Geo 5.3 Medians and Altitudes, Day 2

Lesson Objectives:

- Students will be able to identify special segments in triangles and understand theorems about them.
- Students will be able to find and use the point of concurrency of the medians of a triangle to solve problems and prove relationships in triangles.
- Students will be able to find the point of concurrency of the altitudes of a triangle.

Essential Understanding:

- The medians of a triangle are concurrent at its centroid. The lines containing the altitudes of a triangle are concurrent at its orthocenter.

THEOREM 5-8 Concurrency of Altitudes

The lines that contain the altitudes of a triangle are concurrent.

If...

Then... \( \overline{KQ}, \overline{LN}, \) and \( \overline{MP} \) are concurrent at \( X \)

PROOF: SEE LESSON 9-2.

X is the orthocenter. X is the orthocenter. X is the orthocenter.

Center of gravity!

Ex: Locate the orthocenter

The orthocenter is the point of concurrency of the altitudes of a triangle. How does the type of triangle (acute, obtuse, right) relate to the location of the orthocenter?

Altitudes can be inside a triangle, outside a triangle, or on a side of the triangle.
Ex: Find the orthocenter of a triangle

Where is the orthocenter of $\triangle KLM$ located?

1. $m_{KL} = \frac{5}{10} = \frac{1}{2}$
   $m_{KM} = -2$
   
   $y - 12 = -2(x - 3) \Rightarrow y - 12 = -2x + 6$
   $y = -2x + 18$

2. $m_{ML} = -\frac{7}{7} = -1$
   $m_{LM} = 1$
   $y - 0 = 1(x - 0) \Rightarrow y = x$

   $A$: orthocenter at (6, 6)

Find the orthocenter of a triangle with vertices at each of the following set of coordinates: $(0, 0), (10, 4), (8, 9)$.

1. $M_{AB} = \frac{4}{10} = \frac{2}{5}$
   $y - 0 = -\frac{10}{4}(x - 8)$
   $y = -\frac{10}{4}x + 20$
   
   $M_{BC} = -\frac{5}{2}$
   $y - 0 = -\frac{7}{2}(x - 0)$
   $y = -\frac{7}{2}x$

   $A$: $(10, 4)$

What are the coordinates of the orthocenter of $\triangle KLM$?

1. $m_{JK} = -\frac{4}{6} = -\frac{2}{3}$
   $m_{LJ} = \frac{3}{2}$
   $y - 15 = \frac{3}{2}(x - 11)$
   $y = \frac{3}{2}x - \frac{3}{2}$
   $2y = 3x - 3$

2. $m_{KL} = \frac{15}{5} = 3$
   $m_{JL} = -\frac{1}{3}$
   $y - 4 = -\frac{1}{3}(x - 0)$
   $y = -\frac{1}{3}x + 4$
   $3y = -x + 12$
   $x = -3y + 12$

3. $2y = 3(-3y + 12) = 3$
   $2y = -9y + 36$
   $11y = 36$
   $y = 3$
   $3 = -\frac{1}{3}x + 4$
   $\frac{1}{3}x = 1$
   $x = 3$

$A$: (3, 3)
Essential Question: What are the properties of the medians of the a triangle? What are the properties of the altitudes in a triangle?

- Medians → intersection of midpts (= parts) (centroid)
- Altitudes → intersection of heights (center of gravity)

Vocabulary: The prefix ortho- means “upright” or “right.” How can this meaning help you remember which segments of a triangle have a point of concurrency at the orthocenter?

Height formed w/ right ∠s

Error Analysis: A student labeled P as the centroid of the triangle. What error did the student make? Explain.

P is the intersection of
X bisectors → incenter, not centroid

Reason: Why is an orthocenter sometimes outside the triangle but a centroid is always inside?

- Obtuse Δs can have altitudes outside the figure.
- Centroids are always inside a figure b/c medians are always inside

Look for relationships: Consider the three types of triangles: acute, obtuse, and right. What is the relationship between the type of triangle and the location of the orthocenter? Does the type of triangle tell you anything about the location of the centroid?

- Centroids are always inside
- Orthocenters
  - Acute - found inside
  - Obtuse - found outside
  - Right - found on the rt. ∠
Centers of Triangles Review

Complete the following sentences.

1. The three **perpendicular bisectors** of the sides of a triangle intersect at the circumcenter.

2. The three **angle bisectors** of the angles of a triangle intersect at the incenter.

3. The three **medians** of a triangle intersect at the centroid.

4. The three **altitudes** of a triangle intersect at the orthocenter.

5. A(n) **circumcenter** is equidistant from the vertices of a triangle.

6. A(n) **incenter** is equidistant from the sides of a triangle.

Based on the markings, classify each center as a circumcenter, incenter, centroid, or orthocenter.

1. incenter

2. orthocenter

3. centroid

4. circumcenter

5. circumcenter

6. centroid

Homework: 5.3 Worksheet – Math Literacy and Vocabulary + Additional Practice

Quiz on Thursday!