5.2 Day 2: Bisectors in Triangles

Lesson Objectives:

- Students will be able to prove that the point of concurrency of the perpendicular bisectors of a triangle, called the circumcenter, is equidistant from the vertices.
- Students will be able to prove that the point of concurrency of the angle bisectors of a triangle, called the incenter, is equidistant from the sides.

Essential Question:

- The perpendicular bisectors of the sides of a triangle are concurrent at its circumcenter. The angle bisectors of a triangle are concurrent at its incenter.

#4a) Use a straightedge to draw an acute triangle, \( \Delta ABC \), with sides at least one inch long each.

#4b) Construct the angle bisector of each angle of the triangle. Mark the intersection of all three angle bisectors with point \( P \).

#4c) Construct a circle centered at \( P \) with a radius length equal to shortest distance to one side of the triangle.

#4d) What can you say is true about the radii of the circle?

\( \text{radii are } \perp \text{ to the sides of } \Delta ABC \Rightarrow \text{are } \equiv. \)
#5a) Use a straightedge to draw an obtuse triangle, \( \triangle ABC \), with sides at least one inch long each.

#5b) Construct the angle bisector of each angle of the triangle. Mark the intersection of all three angle bisectors with point \( P \).

#5c) Construct a circle centered at \( P \) with a radius length equal to shortest distance to one side of the triangle.

#5d) What can you say is true about the radii of the circle?

\[ \text{radius should be } \perp \text{ to the sides of } \triangle ABC \text{ and should be } \equiv. \]
#6a) Use a straightedge to draw a right triangle, ΔABC, with sides at least one inch long each.

(b) Construct the angle bisector of each angle of the triangle. Mark the intersection of all three angle bisectors with point P.

#6c) Construct a circle centered at P with a radius length equal to shortest distance to one side of the triangle.

#6d) What can you say is true about the radii of the circle?

- radii should be ⊥ to the sides of ΔABC.
- Is should be ⊥.

Theorem: Concurrency of Angle Bisectors

TDP: Relationships in Δs.
The angle bisectors of the angles of a triangle are concurrent at a point equidistant from the sides of the triangle.

\[ xP = yP = zP \]

Vocabulary:
- The incenter is the point of concurrency at the angle bisectors of a triangle.
- The inscribed circle is centered at the incenter, and the sides of the triangle touch the circle.
Practice:

- The **angle bisector** of the angles of a triangle intersect at a point called the incenter.
- The incenter is always equidistant from the **sides** of the triangle.
  a.) List the angle bisectors: \( \overline{BP} \), \( \overline{CP} \), \( \overline{AP} \)
  b.) Name the incenter: \( P \)
  c.) List all congruent segments: \( \overline{EP} \equiv \overline{EP} \equiv \overline{FP} \)

1. If \( P \) is the incenter of \( \triangle JKL \), find each missing measure.
   \[ PK = \sqrt{8^2 + 17^2} \approx 18.8 \]
   \[ NK = \sqrt{18.8^2 - 8^2} = 17 \]
   \[ LO = \sqrt{13^2 - 8^2} \approx 10.2 \]
   a) \( WP = 8 \)
   b) \( NK = 17 \)
   c) \( PK = 18.8 \)
   d) \( LO = 10.2 \)

2. If \( G \) is the incenter of \( \triangle ABC \), find each missing measure.
   \[ GD = \sqrt{37^2 - 35^2} = 12 \]
   \[ BG = \sqrt{6^2 + 12^2} \approx 13.4 \]
   \[ FC = \sqrt{3^2 - 12^2} = 15 \]
   a) \( GD = 12 \)
   b) \( BG = 13.4 \)
   c) \( FC = 15 \)
   d) \( BF = 6 \)

3. If \( S \) is the incenter of \( \triangle PQR \), \( m \angle QPR = 8x - 10 \), \( m \angle RPQ = 4x - 14 \), and \( m \angle PQR = 7x - 5 \), find each missing measure.
   \[ 19x - 29 = 180 \]
   \[ x = 11 \]
   a) \( m \angle PRQ = 78^\circ \)
   b) \( m \angle RPQ = 30^\circ \)
   c) \( m \angle PQR = 72^\circ \)
   d) \( m \angle RPS = 15^\circ \)
   e) \( m \angle PQS = 36^\circ \)
   f) \( m \angle PRS = 39^\circ \)
   g) \( m \angle PSR = 126^\circ \)